The thesis entitled **"Group theoretic similarity solutions of Diffusion Equations"** is an outcome of author's investigations on theoretical aspects of the different kind of problems of diffusion. It demonstrates the use of Group theoretic similarity solutions of various types of problems associated with diffusion arising in engineering and sciences. It contains six chapters, out of which the first two are devoted to the general introduction of the subject, its Mathematical development and Physico-Mathematical background. The author's investigations on the diffusion problems are included in subsequent three chapters.

In most of physical phenomena Mathematical modeling leads the formation of either ordinary differential equations or partial differential equations. Sometimes leads to a set of simultaneous systems of equations. Ideally one hopes to find the exact solutions of these equations, but it is well known fact that there is hardly a full proof method for the treatment of all such equations. Researchers are using various methods like analytical method, numerical method, etc to find closed form solutions of these equations. The concept of Mathematical similarity analysis has a great role in the basic simplification of the analysis of such methods. We have used the group theoretic similarity techniques for solving the problems of diffusion.

Chapter 1 represents a sound structure in logical manner to provide knowledge of very basic concept of diffusion. Diffusion is useful to engineers, chemists, life scientists and to our daily lives. This chapter deals also with definition and brief discussion of four fundamental laws introduced by: (1) Thomas Graham, (2) Adolf Fick and (3) Darcy and (4) Fourier. In addition it contains some useful information about the rheological development of these laws. The wonders of diffusion are in general represented by the partial differential equations.

Chapter 2 contains the discussion about the group theoretic similarity techniques in general. Today there are number of similarity techniques found in literature out of which (1) Free Parameter, (2) Separation of Variables, (3) Group Theoretic, (4) Dimensional Analysis are frequently used. Among these methods, Group Theoretic methods based on finite transformation as well as infinitesimal transformation are more attractive than the other methods, because these methods derived systematically from simple group transformation laws. The various application of different similarity techniques are given by Hansen and his co-workers. Today, in the modern age, the use of similarity analysis is not restricted to fluid–mechanics or heat transfer, but it is much useful in studying physical, biological, medical and

social sciences. In this chapter we discuss the finite transformations and infinitesimal transformations for the partial differential equations governing the diffusion. Lie has introduced the study of continuous group of transformations of ordinary differential equations based on the infinitesimal properties of the group.

Chapter 3 deals with the problems of one-dimensional diffusion equation associated with heat equation. We begin by the formulating the equations of heat flow describing the transfer of thermal energy. Heat energy is caused by the agitation of molecular matter. Two basic process conduction and convection take place in order for thermal energy to move. We study heat flow only in cases in which the conduction of heat energy is much more significant than its convection. We thus think of heat flow primarily in the case of solids, although heat transfer in fluids is also primarily by conduction if the fluid velocity is significantly small. We derive the equation of the conduction of heat in one dimensional rod by considering the heat energy. The heat energy contained in a region is defined to be the energy it takes to raise the temperature. We can relate this heat energy to the temperature by use of specific heat. This physical phenomenon using ‘integral conservation law’ and the ‘Fourier law’ is represented by the second order linear partial

differential equation which is known as the heat equation-diffusion equation, using the finite as well as infinitesimal transformations of group techniques, the obtained partial differential equation is converted into the second order nonlinear ordinary differential equation. The exact solution of the later equation is obtained by using classical method for solving the second order ordinary differential equation.

Chapter 4 discusses about the derivation of one-dimensional partial differential equations, which are associated with diffusion process. It includes the nonlinear burger's equation. The infinitesimal transformations in searching for all possible groups of transformations to these partial differential equations are developed. The procedure is based on Lie's theory of finite and infinitesimal continuous group of transformations. This group theory has been applied extensively in recent times by investigators in the field of similarity analysis. By this method, these partial differential equations are reduced to the ordinary differential equations. The obtained ordinary differential equations are solved by the analytical methods.

Chapter 5 concerns with the discussion of miscible displacement. In this chapter, discussion of longitudinal dispersion, this is the process by miscible fluids in laminar flow mix in the direction of the flow. The

phenomenon is discussed by regarding the cross-sectional flow velocity as time dependent in specific form. The mathematical formulation of the phenomenon using Darcy's law yields a nonlinear diffusion equation. This partial differential equation is transformed into ordinary differential equation using similarity analysis. An analytical solution of the later is obtained in terms of confluent hyper-geometric functions.

In conclusion, we have obtained the analytical solutions for the equations associated with the diffusion using finite as well as infinitesimal group transformation. In the present work the author has obtained similarity solution of the governing linear as well as nonlinear diffusion equations. Thus to sum-up the subject matter of the thesis adds to the existing literature on the investigated topic by making some original contributions giving fresh interpretation to known facts and extending the scope of Mathematical study.

Lastly, it may be mentioned that the author has been greatly influenced and encouraged by the excellent work of Scheidegger A.E., Hansen A.G., Blueman G.N., Cole J.D. and Murphy G.N.

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